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A Model of the Russian Crisis Development

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This study focuses on the 'hostage effect,' which enables a government to share the burden of crisis prevention with the private sector. In the most severe situations this mechanism turns out to be the only way for the government to mitigate the crisis. It is demonstrated that the crisis model accounting for the 'hostage effect' implies reversed logic of coordination. As a consequence, standard approaches to curing crises may produce results opposite to those predicted by common sense. We find in particular that the more reserves the government has in this model, the stronger is the adverse effect of the crisis. The sources of the 1998 financial crisis in Russia are discussed. We argue that some of the effects revealed by our model could contribute to the development of this crisis. The model can explain, for instance, the apparently adverse impact of loans provided by the IMF and the World Bank.

Keywords. Russia, financial crisis, coordination failure, bail-out.

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CONTENTS

1. INTRODUCTION

The recent events in Argentina and Turkey provide evidence that the stability of world economies is getting more and more fragile. Contemporary in-depth positive analysis of crises sources and mechanisms is now paralleling efforts to formulate the new principles of the international financial system.

We discuss below an effect that may play an important role in the development of the most severe crises. This effect may appear in situations where the capital market is dominated by a few large players and the government is lacking liquidity to withstand temporary deterioration of the external environment or a speculative attack.

The key point of the effect is the government's possibility to share the burden of fighting the crisis hazard with the private sector. This may be possible as creditors who are holding government assets or assets denominated in the national currency have additional incentives to prevent a crisis. The government can use these assets as 'hostages,' making investors support its fight against the crisis. In a sense this is a model of the possible cooperation between the government and private sector in mitigating the financial crisis. But as the government forces investors to bail it out, the effect under consideration may be important only in the most severe crisis cases.

The 'hostage effect' incorporates the problem of coordination, as do most second-generation models. But it is demonstrated that the logic of coordination is reversed here. Standard models imply that supporting the *status quo* becomes more attractive for an investor if other participants also decide to support the *status quo*. When the government makes use of the 'hostage effect', increased (for some reasons) efforts by some investor to support the *status quo* lead to lower efforts by other participants. As a consequence, standard approaches to curing crises may produce results opposite to that predicted by common sense.

We argue that some of the effects revealed by our model could contribute to the development of the financial crisis in Russia in 1998. This concerns first of all the apparently adverse impact of loans provided by the IMF and the World Bank. Contrary to the standard thinking, their support not only failed to relieve the crisis, but rather precipitated the collapse, which occurred almost immediately after receiving these loans.

2. CHARACTER OF THE RUSSIAN FINANCIAL CRISIS

The disputes on the nature and specifics of the Russian crisis of 1998 have mainly abated, and a comprehensive analysis of the crisis development has been published (see for instance Kharas, Pinto, and Ulatov, 2001), but still fully convincing answers to many key questions are missing. Views on the origins of the Russian crisis vary widely. Some analysts (for example, Alexashenko, 1999) argue that Russia has experienced mainly a debt crisis, which developed as a result of the soft fiscal policy carried out by the government. It is often asserted that the GKO market was, in fact, a Ponzi scheme, and its collapse was initially inevitable and only slightly precipitated by the Asian crisis. According to this view, the debt crisis aroused a currency crisis, which otherwise would not have occurred, as the Central Bank (CBR) implemented tight monetary policy. The opposite view (presented for instance by Montes and Popov, 1990) is that Russia experienced a currency crisis caused by strongly distorted targets of the exchange rate policy (significantly overvalued ruble), while the debt crisis was not inevitable and happened due to the erroneous measures of the authorities. Kharas, Pinto and Ulatov (2001) and Perotti (2001) stress soft budget and legal constraints as a fundamental source of the crisis. We argue that the explanation of both its origin and course is more complicated, involving several components.

We consider the same general factors as other authors, yet our conclusions differ somewhat from the common views. One of our main points is to show that when discussing the Russian crisis, one should make a thorough distinction between the situation before and after the commencement of the crisis. Assertions about the sustainability of macroeconomic policy, which are often made without exact specification of timing and time horizon, mislead rather than explain the crisis causes.

Before turning to our analysis, let us look at the economic developments on the eve of the crisis. The government bond (GKO/OFZ) yields were rapidly falling in 1997. Yields for 6-month GKO's dropped from 45% in December 1996, down to 16–17% in July–October 1997. Ex-post real rates fell to only 8% in the Q3 1997. This decline was to a great extent explained by the participation of non-resident investors. Gurvich and Dvorkovich (1999) estimate the contribution of the integration of the GKO market into international capital markets as 2/3 of the total observed decline.

Lower interest rates resulted in some slowing down of the debt stock: its growth amounted to 84% in 1997, as compared to 209% in 1996. Domestic debt of the federal government increased in 1997 by 3 percentage points of GDP, as compared to 5 percentage points in 1996, though

borrowing by the government still remained quite substantial: total GKO/OFZ's placement was equivalent in 1997, as in 1996, to 20% of the GDP. The duration of domestic debt was gradually growing, although still remaining quite short, not exceeding 1 year. This fact determined the enormous size of the current debt redemption due: GKO/OFZ's redemption due in 1998 was over 1.5 times higher than the current revenues of the Federal budget, this being one of the key causes of debt crisis.

At the same time the transformation recession ceased and production recovery began. The growth rate was relatively high during this period, amounting in Q2–Q4 1997 to some 4% in annual terms. In other words, investors' confidence, fuelled with expectations of a forthcoming recovery, was increasing during this period despite the remaining imbalanced fiscal policy of the government.

We agree with Trofimov (1999) that the GKO market should not be viewed as a Ponzi scheme at that time. A simple analysis of debt sustainability reveals that substantial, but quite feasible modifications in the fiscal policy were needed to stabilize the debt ratio to GDP (which was not too large at that moment). Hence debt policy was not sustainable in the long run, but could be made sustainable by moderate amendments. At the same time, falling interest rates prove that it was viewed as sustainable in the short run by investors.

A salient feature of the Russian GKO/OFZ market was the domination of large players. As pointed out by Medvedev and Kolodyazhny (2000), the leading role in the market was played by a limited number of dealers and non-residents. The government accounted for this fact in the course of the crisis by trying to discuss measures to prevent failure with major investors.

The next important question is: was ruble overvalued? First of all, UN estimates of the relationship between exchange rates and PPP provide evidence that this ratio for the ruble (43%) was roughly equal in 1996 to that for the Czech (43%) and Slovak (40%) currencies, and was lower than that for the Hungarian (48%), and Polish (51%) currencies. Hence, according to these estimates the ruble was not overvalued in comparison with the currencies of other transition economies. The same conclusion can be obtained from a comparison of the wages in these economies in dollar terms. Similar conclusions can be derived from the main test of 'exchange rate appropriateness': the sustainability of the balance of payments. On the one hand, Russia's balance of payments was supported by a large-scale inflow of short-term capital: in 1997 new investment to GKO's amounted to $11 billion, external borrowing by the government made up another $11 billion, inflows to the private sector turned

to be $24 billion, of which direct foreign investment accounted for only $6 billion. Thus the capital inflow totaled $45 billion, with almost 2/3 falling on short-term investment, and we admit that such huge inflows exceeded by far their normal level. But, on the other hand, it should not be forgotten that capital outflow from Russia (in the most broad sense, as the increase in foreign assets of the private sector) was almost as large, reaching $40 billion in 1997. Besides, capital inflows and outflows were highly correlated during the period under consideration, *i.e.*, the former were often used as a source for the latter.

According to our analysis, real exchange rate sustainable in the medium term equals about 80% of its pre-crisis level. In other words, the exchange rate was overvalued, but not so much: reaching a sustained exchange rate required real depreciation by only 25% instead of 100%, as it actually occurred in the H2 1998. Surely, capital outflow would hardly reverse as fast as short-term investments; still this proves that in the medium term only moderate modification of the exchange rate policy was required.

This situation was broken abruptly in late 1997, when the world commodity prices dropped. As a result export value fell by $15 billion in 1998, though its volume slightly increased. Sharp deterioration of the BOP made the then effective exchange rate no longer sustainable even in the short run, and nobody could know how greatly this situation would deteriorate in terms of trade, and how long it would last. The monetary authorities had two options: to abandon the 'crawling peg' exchange rate regime and switch to a floating exchange rate policy, or to defend ruble. The Central Bank and the government chose the latter option.

It is clear that success in defending the ruble depended critically on the duration of the crisis: the actual rate could be maintained only in the very short run. Hence the actual policy choice made implied **quite optimistic** expectations of a rapid recovery of commodity prices. Throughout the crisis period, authorities were arguing that prices would recover in some 3 or 4 months, and their actions were based on this presumption. The major measures included:

- replacing short-term domestic borrowing with long-term external borrowing,
- swapping of the GKO/OFZ's debt falling due in 1998–1999 (worth 27 billion rubles, or the equivalent of around one month of redemption due) for long-term eurobonds (worth $5.9 billion),
- requesting an urgent IMF loan,
- reducing domestic borrowing (net domestic financing had been negative since March 1998) by cutting spending at the cost of building up arrears in the budget.

Major measures by the government amounted to efforts to cut short-term need in liquidity and to obtain additional liquidity to survive the transient (as it was assumed) period of unfavorable external conditions.

The elaboration of the Fiscal Consolidation Program was a more fundamental policy measure, but it faced difficulties in passing the Duma and as a result was not implemented.

A combination of all these circumstances resulted in the development of two parallel, very fast and closely interrelated processes:

- Increased demand for hard currency,
- Falling demand for ruble-denominated government debt. Average GKO yields hiked to 37% in December 1997, 24% in March 1998, 55% in May 1998, and 81% in July 1998.

The underlying mechanism, as we see it, was the following.

- **Expected depreciation raised GKO interest** rates via the 'interest parity ratio'. It is important to draw attention to the fact that while yields for 6-month GKO's hiked from 16–17% in the Q3 1997 to 31–32% in the H1 1998, yields for comparable MinFin bonds increased relatively slightly: from 8–9% to 9–12%. It should be noted that despite the commonly held view, the change in investors sentiments towards emerging markets **had nothing to do with the lower demand for GKO's**, as the share of non-residents in the GKO/OFZ market (as well as non-resident's holdings in dollar terms) was growing during the crisis.
- Borrowing at such rates evidently made fiscal policy unsustainable, hence the government made efforts to cut domestic financing as noted above. Domestic public debt has increased in the first half of 1998 by only 16%.
- Free money was directed then to the currency market, **depleting the Central Bank's international reserves** (the latter fell from $23 billion in the end of October 1997 to $15 billion in the end of March 1998),
- The Central Bank, trying to defend the ruble from devaluation, **used to raise refinancing rate**,
- High interest rates in the money market relaxed pressure on the ruble for a while, but, on the other hand, they supported high interest rates on GKO's market, and **suppressed production**. Industrial output dropped in 9 months of the crisis period (from October 1997 to July 1998) by 10% on a seasonally adjusted basis.

It is clear that such a process could be sustained for only a very short time, and the outcome depended on how long the period of low commodity prices may last. This was a gamble: monetary authorities bet that prices will start recovering before reserves are depleted. And if this had really occurred a year earlier than it happened, the crisis could well have been evaded in the short term, and perhaps it would make the government modify its policy and thus evade the crisis in the long run as well.

Summarizing, we can conclude that the macroeconomic policy pursued by the monetary authorities was not robust in the medium run, but, in the absence of external shocks, it was far from a crisis area, and required moderate, feasible modifications to be viable. The impetus to the crisis was given by a sharp deterioration in terms of trade. After this shock the previously pursued policy was no more sustainable in the short run and required drastic changes. First of all, switching to a floating exchange rate was urgently needed. The authorities underestimated the scale of deterioration in the fundamentals, and failed to make adequate adjustments to the policy. They assumed that 'bad times' would finish shortly, and addressed the situation as a transient liquidity crisis. As optimistic expectations did not realize, the crisis became inevitable. The debt market was not the source of the crisis, but was the weakest, most vulnerable element hit by the crisis. The fundamental cause of the immense yields on the GKO market was the expectation of devaluation, not distrust of the Russian government's debt or of emerging market securities in general.

After the crisis, the government had to make fundamental modifications in its macroeconomic policy, but this does not prove by itself that the previous policy was doomed to fail. The course of events only confirmed the necessity to react adequately to serious changes in the fundamentals.

The following salient features of the Russian financial crisis underpin parallels with the model presented below.

- The crisis was ignited by unexpected external shock,
- Prevalence of short-term domestic debt made the government dependent on the investors' decision to roll-over the debt, as debt redemption due exceeded by far the current fiscal revenues,
- The debt and currency crises developed in parallel, as investors mainly moved resources between these two markets.
- The monetary authorities viewed the problem primarily as a lack of liquidity, and correspondingly addressed it mainly by trying to get additional external financing (to increase both the foreign currency reserves and the fiscal reserves).

- Another important way of fighting the crisis was raising interest rates to make ruble assets more attractive.
- The GKO/OFZ market was dominated by large players.
- Failure to prevent the crisis resulted in substantial losses borne by GKO/OFZ holders. Their total losses (taking into account both the change in face value and in real value as a result of inflation and depreciation) are estimated as almost 90%.

3. LITERATURE REVIEW

Many aspects of the problems addressed in this paper are related to those considered in other publications on crises modeling.

Many authors discuss the problem of defending an exchange rate from attack by raising interest rates. Bensaid and Jeanne (1997) incorporated into the model of exchange rate crisis costs of raising interest rates (due to the increase in public debt, negative impact on economic activity, weakening of the banking system, *etc*.). They found that awareness of these costs by speculators may generate a self-fulfilling crisis and provided informal evidence that that their model can explain some features of the 1992–1993 EMS crisis. Lahiri and Vegh (2000) demonstrated that it is feasible to delay a BOP crisis by raising interest rates, but this policy has intrinsic limitations. Raising rates above a certain level may actually hasten the crisis. In addition they prove that there is an area where monetary authorities can use interest rates to prevent a crisis, but the costs outweigh the gains. Flood and Jeanne (2000) come to the conclusion that raising interest rates makes domestic assets more attractive but weakens the domestic currency by increasing the government's fiscal liabilities. As a result, a speculation attack motivated by underlying fiscal fragility may be hastened by raising interest rates.

The next subject addressed in many models is the role of liquidity constraints. The key point in this discussion is that low liquidity may lead to a self-fulfilling crisis because of coordination failure among creditors: the country will service its debt or default, depending on decisions by investors to roll over the debt or to refrain from it. Related issues are how to tell insolvent debtor from illiquid ones, and moral hazards arising from extending liquidity. Some of the recent results in this field can be summarized as follows. Bussiere and Mulder (1999) find that higher liquidity can offset weak fundamentals and limit the vulnerability of countries in the period of contagion. They argue that liquidity is required while

investors learn whether the debtor hit by the crisis is solvent or not. Empirical analysis carried out by Detragiache and Spilimbergo (2001) shows that the probability of a failure in foreign debt service depends significantly on external liquidity after controlling for the debt structure and for macroeconomic variables. Many papers examine reasonable reserve thresholds. Obstfeld (1986) demonstrated that for a country that can borrow from international capital markets, foreign reserves can become infinitely negative without violating the government intertemporal budget constraint. Disyatat (2001) constructed a second-generation model that reflects limited borrowing at a cost as well as the macroeconomic costs of defending the exchange rate. In the framework of this model, a crisis may occur due to inadequate reserves, and the optimal amount of reserves can be estimated.

Many authors (for instance, Feldstein, 1999) argue that liquidity is the key to self-protecting countries (especially emerging markets) from crises. In cases when a country loses access to capital markets, international institutions may play a role of a 'lender of last resort' in preventing a crisis (Fisher, 1999).

Many papers investigate the possible role of large players as sources of market destabilization. Corsetti, Pesetti and Roubini (2001) discuss analytical results from different models suggesting that the presence of agents with high market power can increase a country's vulnerability to a crisis.

Finally, our study has some common features with papers on the role of soft budget constraints in banking crises (Dewatripont and Maskin, 1995; Mitchell, 1999). These authors also discuss situations where an investor has strong incentives to bail-out the debtor.

4. CRISIS MODEL

Below we examine a simple model of a crisis. Its main features can be summarized as follows. A crises occurs, which originates from an unexpected external shock (say, contraction of fiscal revenues due to falling export prices). Authorities, as a result, experience a shortage of liquidity to support the *status quo* (keeping the exchange rate regime, or servicing public debt). If we are talking about debt crisis, the origin of a crisis is the possibility that resources available to the government may fall short of its commitments, *i.e.*, a fiscal gap may arise. We assume that the government is unable to fully cover this gap by increasing fiscal revenues and/or cutting expenditures, and hence considers additional borrowing to cover the financial gap.

Both future revenue and the level to which the government is willing and able to cut its commitments are uncertain. If new investment happens to be insufficient to honor all payments due, the government has to cover the gap by printing money, or to reduce commitments by defaulting. The result is, on the one hand, partial loss in value of the debt held by investors (debt failure), and on the other hand, losses for the government (due to an adverse effect on investment and production, undermined confidence to the monetary authorities, *etc.*). At the same time our model also accounts for the positive effect of a crisis for the government which gains from alleviating the burden of public debt.

It is natural to assume that the degree of losses borne by debt holders positively depends on the amount of the uncovered fiscal gap. This assumption looks especially plausible, if we are talking about domestic debt and assume that the most probable way of covering the gap, if the government resources are insufficient, is printing money. In this case depreciation of the debt depends on inflation, which depends on the relationship between the initial money supply and additional emission. This, in turn, implies their negative dependence on the size of new lending to the government.

The problem is that if the government is under risk, a relatively high yield should be suggested to attract investment (at least within the standard logic). The government has to trade off the disadvantages of the crisis with the costs of fighting it (which include both direct costs of borrowing and indirect adverse effects of raising interest rates in the economy). In addition, we should take into account that the crisis has not only adverse, but also some positive effects from the viewpoint of the government, as it alleviates debt burden. The first question to be answered by our study is under what conditions the government has a chance to pay acceptable costs for reducing the risk of a crisis.

An important point of our study is to account for the fact that not only the government is interested in evading a crisis, but so are investors holding debt under risky circumstances. Lending to the government, on the one hand, increases investor's assets under risk. On the other hand, lending reduces the possibility of a debt crisis and hence the expected losses from it for the debt holder. Additional incentives for debt holders to invest can change the situation critically if the government decides to incorporate these incentives into its anti-crisis policies.

Normally higher debt increases the risk of not servicing it fully and/or in time by the government. But in the situation of an acute crisis, additional borrowing reduces immediate risks of a liquidity crisis. Still it is clear that the side effect of new borrowing is an increased chance of a debt crisis

in the long term. On reaching some level, the latter effect becomes dominating: indeed if the debt stock becomes too large, it is impossible for the government to pay it off. We assume hence that the rate of loss is decreasing for not too large amounts of investment, and growing for larger amounts. The first interval reflects the short-term effects, while the second one is responsible for the long-term effects.

It is important to note that, generally, the stock of the public debt affects the size of the government's commitments, and hence, everything else being fixed, is linked to the size of the fiscal gap. But, on the other hand, the fiscal gap depends on relations between budget revenues, non-interest commitments, and debt payments due (including interest payments and debt redemption). The latter, in turn, depends on the maturity of the debt. In other words, debt stock is only one of the factors, which taken together define the amount of the fiscal gap. Being interested in the analysis of the 'hostage effect', we compare debt crisis outcomes in situations with different values of debt stock but with the same characteristics of a fiscal gap.

We are dividing decisions concerning new lending into two parts, assuming that the government is first choosing the interest rate suggested to investors, and then the latter are choosing the amount they are willing to lend at this rate.

The effect of borrowing is ambiguous for both investors and the government. Common sense tells us that to attract investment during a crisis, the government has to suggest a high interest rate, compensating for the risk of losses. This, first, raises future interest payments, and second, has adverse effect on production by suppressing credit available to the economy. Thus, making decisions about investment, the government has to compare the lower risk of the crisis with the costs of borrowing and debt alleviation if the crisis occurs, and investors have to take into account the positive short-term and negative long-term impacts of lending in a time of crisis risk, and to compare both with the effect of an increased amount of assets under risk in their portfolios.

Next, we have to formulate criteria for decision-making by the government and investors. The natural assumption is that the payoff of the government negatively depends on the interest payments to investors, and positively depends on the loss of real value of the public debt (as it relieves debt burden). In addition, the government's payoff function should reflect two indirect effects. The first is debt failure, which has an adverse impact on the economy. Our assumption is that it is reflected in the government's payoff as a negative function of the rate of losses borne by investors. Second, raising interest rate hits production and fiscal revenues, hence the pay-off function negatively depends on the interest rate.

Our model also has alternative interpretations. First, it can be equally viewed as a model of a currency crisis. In this case investors decide whether to invest in the foreign currency to save their cash from depreciation or to support the national currency to evade depreciation of domestic assets which they initially hold.

Another interpretation can be given in terms of project organization theory. Investors decide whether or not to bail-out the project under risk to save their potential return on their previous investment. Both alternative interpretations require no modifications of the model.

Now we can give the formal specification of the game. It has $N + 1$ participants (the government and N investors) and is characterized by parameters $d_i \geq 0$ (government debt held by i-th investor) and r_0 (yield for non-risk investment, an alternative to lending to the government). Important characteristics of the game also include the following:

- Loss function $\omega(X)$, which is falling for all $X < H$, and growing for $X > H$, with $\omega(X) \to 1$ as $X \to \infty$,
- Minimal rate of losses $\omega_m = \omega(H)$,
- $\varphi(r)$ — growing with r function, reflecting the adverse impact of raising interest rate r on the economy,
- $Z(\omega)$ — growing function of ω, reflecting the adverse impact of debt failure on the economy (the larger is the rate of losses ω, the stronger is this effect).

All functions are assumed to be continuous and twice differentiable. In addition, we assume that ω is concave and its derivative has limited value in a zero point: $\omega'(0) \neq -\infty$.

The game has 2 stages.

Stage 1. The government chooses yield r suggested to investors or decides not to borrow.

Stage 2. All investors independently choose amounts x_i of lending to the government at the interest rate r. If the government decided in the first stage not to borrow, it is assumed that $x_i = 0$ for all i, and $r = 0$.

The payoff function of i-th investor w_i is defined as an expected change in his assets resulting from his and others' actions:

$$w_i(\{x_i\}, r) = x_i (r - r_0) - (d_i + x_i + x_i r)\ \omega(X),$$

where $X = \Sigma\, x_i$ is the total amount of lending provided by all investors. The first component reflects additional interest received on lending

(compared with investing in a risk-free asset), while the second one reflects losses from debt failure.

The government's payoff also consists of two parts:

$$V(X, r) = -[X(r - r_0) - (D + X + X r)\,\omega(X)] - [\varphi(r) + Z(\omega(X))]$$

(D being total amount of debt initially held by investors: $D = \Sigma\, d_i$).

The first part equals the sum of investors' payoffs taken with an inverse sign (as both interest gains by investors and their losses in the case of debt failure correspond to equal losses and gains by the government). The second part reflects the adverse effect φ of raising the interest rate and the adverse effect of a debt failure Z.

Our objective is to identify solutions of this game, to analyze how large investment can be and what defines its magnitude. The amount of new investment in this model defines the degree of the crisis and hence can be viewed as the level of coordination between the government and investors in preventing a crisis. We shall call the game solution where total investment is zero 'trivial,' and that with positive total investment 'cooperative.'

5. GAME SOLUTION

First, we will prove the existence of the game solution.

Proposition 1. If: 1) portfolios of investors are large enough and not very different in their sizes, 2) the adverse effect of a crisis is relatively strong ($Z'(\xi) > D$ for all $\xi \in [0, 1]$), then the game has a single equilibrium, which is positive.

Proof. Suppose that interest rate r is fixed. The lending decisions taken by investors under fixed r can be regarded then as a sub-game. The solution to this sub-game (if one exists) is called positive, if $x_i > 0$ for all i.

The positive equilibrium of the sub-game of N investors with portfolios D_i is a set $\{x_i^*\}$, $x_i^* > 0$, that satisfies the following conditions:

$$\left\{ x_i^* = \arg\max_{x_i} W_i(X_{-i}^*, x_i),\ i = 1, \ldots, N,\ X_{-i}^* = \left\{x_j\right\}_{j \neq i} \right.$$

The pay-off function of the i-th investor can be presented as

$$w_i(\{x_j\}, r) = x_i \rho(X) - d_i \omega(X),$$

where $\rho(X) = r - r_0 - (1+r)\omega(X)$.

The first-order condition of the maximization problem has the following form:

$$\{w_i'(\{x_j\},r) = \rho(X) + x_i\rho'(X) - d_i\omega'(X) = 0,\ i = 1, \ldots, N. \quad (1)$$

From this system one immediately derives that if $\omega'(X) \neq 0$, $(1+r)x_i + d_i = (1+r)x_j + d_j,\ \forall i, j$.

Using this last equation, the first-order condition can be transformed into a system of one differential equation and $N - 1$ algebraic ones. To demonstrate this, let us denote, without loss of generality, $d_1 = \max\{d_i\}$. Then the following is just a simple transformation:

$$x_i = x_1 + \frac{d_1 - d_i}{(1+r)},$$

$$X = \sum_i x_i = Nx_1 + \boldsymbol{F},\ \text{where } \boldsymbol{F} = \frac{1}{(1+r)}\left(Nd_1 - \sum_{j=1}^{N} d_j\right).$$

Now the first-order conditions (1) can be rewritten to show that investment made by participants 2, ..., N are in fact functions of investment x_1 chosen by the first participant:

$$\begin{cases} w_1'(\{x_j\}) = \rho(Nx_1 + \boldsymbol{F}) + x_1\rho'(Nx_1 + \boldsymbol{F}) - d_1\omega'(Nx_1 + \boldsymbol{F}) = 0, \\ x_i = x_1 + \dfrac{d_1 - d_i}{(1+r)},\ i > 1. \end{cases} \quad (2)$$

To show that the system (1) has a non-zero solution, one has to prove that the differential equations have such solutions. Suppose that

$$\left\{ d_i > \frac{r_0 + \omega(\boldsymbol{F})}{-\omega'(\boldsymbol{F})},\ i = 1, \ldots, N \right.$$

(these conditions specify the assumptions introduced above that portfolios are large enough). An important role here is played by the special interest rate r_H:

$$r_H = (r_0 + \omega_m)/(1 - \omega_m).$$

This rate covers possible risks to investors if the rate of losses equals ω_m. Once this is the lowest possible level of losses, r_H represents the cutting point: any yield below r_H will not compensate for the investment

risks and $\rho(X)$ will be negative for any X; while yields above r_H may compensate for the risks or not, depending on the decisions of all investors. Then, as one can easily see, $w_i'(0) > 0$ for any i and any r. Next, $w_i'(X) < 0$ whenever $X \geq H$ for any $r < r_H$. When $r > r_H$, given the properties of the ω function, there exists such large X that $w_i'(X) < 0$. Given that $w_i'(X)$ are continuous, it follows that, if $\boldsymbol{F} < X$, some point $x_i^* > 0$ exists, where $w_i'(x_i^*) = 0$. It can be easily seen that if interest rate r_H is suggested by the government, any set $\{x_j\}$ such that $\sum x_j = H$ is a Nash equilibrium, and no other equilibria exist.

Now we turn to the analysis of the government's choice of interest rate.

The payoff obtained by the government in any of the investors' equilibrium at r_H equals $V(H, r_H)$. The government thus always has the possibility to attract total investment H, and to ensure payoff $V(H, r_H)$.

Next, it can be demonstrated that the government's strategies with $r > r_H$ are dominated by r_H. The proof includes two cases: $X \leq H$, or $X > H$. In fact the former case is impossible, as each participant could gain then from increasing his lending by amount $H - X$. Indeed, let $z^* = x_i + (H - X)$ be new investment by i-th participant. Then

$$w_i(z^*, X_{-i}^*, r) - w_i(x_i^*, X_{-i}^*, r) =$$

$$= d_i(\omega(X) - \omega_m) + x_i^*(\omega(X) - \omega_m) + (z^* - x_i^*)[r - r_0 - (1 + r)\omega_m] > 0,$$

as all components on the right side are positive.

In the case when $X > H$, the following inequalities hold:

$$\begin{aligned} V(r_H) - V(r) &= \\ &= D\omega_m - \varphi(r_H) - Z(\omega_m) + X\rho - D\omega(X) + \varphi(r) + Z(\omega(X)) = \\ &= X\rho - D(\omega(X) - \omega_m) + (Z(\omega(X)) - Z(\omega_m)) + (\varphi(r) - \varphi(r_H)) = \\ &= (Z'(\xi) - D)(\omega(X) - \omega_m) + X\rho + (\varphi(r) - \varphi(r_H)) > 0. \end{aligned}$$

Hence values of $r > r_H$ could be disregarded.

To show that the equilibrium X^* is unique, one has demonstrate that W_1'' is negative for any $y \in [0, H - F/N]$. Together with relations (2) this will lead to the uniqueness of $\{x_i^*\}_{i>1}$ and to the satisfaction of the second order conditions for the solution of the system (2). The second derivative

of the pay-off function of the first player has the form:

$$w_1''(X) = 2\rho(Nx_1 + \boldsymbol{F}) + x_1\rho''(Nx_1 + \boldsymbol{F}) - D_1\omega''(Nx_1 + \boldsymbol{F}).$$

So the equilibrium uniqueness condition has the following form:

$$\begin{cases} \boldsymbol{F} < H, \\ d_1 > \dfrac{-2(1+r_H)\omega'(\boldsymbol{F})}{\omega''(\boldsymbol{F})}, \\ \omega'''(y) > 0, \ \forall y \in \left[0, H - F/N\right]. \end{cases}$$

To complete the proof, one has to show that the conditions of existence and uniqueness of the maximization problem solution are consistent with the individual rationality constraints. Consider whether it is profitable for the *i*-th investor to abstain from investment when the others invest optimal amounts for the *N*-player game. In this case, given the properties of the pay-off functions, one can see that point $\left\{X^*_{-i}, 0\right\}$ is on the left of the solution of system (2). Therefore the pay-off functions at this point are increasing with respect to x_i. So by investing some amount, the *i*-th player will become better off. This means that it is not profitable for the *i*-th investor to deviate from the equilibrium point. Q.E.D.

One of the basic conditions for the existence of the equilibrium in the *N*-player game is the restriction on the differences of portfolio sizes. What happens when this restriction is broken? Obviously a positive equilibrium in the *N*-player game will not exist. On the other hand, this might be the case when $N - 1$ players invest non-zero amounts and the other one withdraws from the game. Is this always possible or not?

Consider the game of *N* investors with $\boldsymbol{F} > H$ and $\boldsymbol{F}_{-i} < H$ for any *i* (where $\boldsymbol{F}_{-i}$ is defined, as above, as the net of the portfolio of the *i*-th player). Suppose that all other conditions for any $(N - 1)$-player sub-game to have a positive equilibrium hold. This equilibrium will be the one of the *N*-player game with the *i*-th investor staying aside if she will not have incentives to enter when the others invest optimal amounts for the $(N - 1)$-player game. Let $\{x^*_{-i}\}$ be the equilibrium of the $(N - 1)$-player game without the *i*-th investor. Consider the first derivative of the *i*-th player's pay-off function at this point.

$$w_i'(\{x^*_{-i}, 0\}) = \rho((N-1)x_1^* + \boldsymbol{F}_{-i}) - d_i\omega'((N-1)x_1^* + \boldsymbol{F}_{-i})$$

If this derivative will be less than 0, the *i*-th player will not enter the game and the equilibrium with $N - 1$ players investing and the *i*-th one not will

exist. This happens if her portfolio is not too large:

$$d_i < -\frac{\rho((N-1)x_1^* + \boldsymbol{F}_{-i})}{\omega'((N-1)x_1^* + \boldsymbol{F}_{-i})}.$$

Now, for each interest rate r ($0 \le r \le r_H$) chosen by the government, its payoff $V^*(r)$ is defined. Limited function $V^*(r)$ reaches its maximum on $[0, H]$. To conclude, it should be noted that the option 'not to borrow' is inferior for the government as compared with suggesting a zero interest rate.

This completes the proof that the game has a unique positive equilibrium. We turn now to the analysis of this equilibrium. Its nature can be seen from the following discussion.

First, if assumptions of Proposition 1 hold, the government always suggests an interest rate below r_H. This follows from the fact that $V_r'(r_\mathrm{H}) = -H(1 - \omega_m) - \varphi'(r_H) < 0$. In other words, the *optimal policy of the government always involves suggesting an interest rate not covering investment risks*, *i.e.*, *resorting to a 'hostage effect'*.

Second, total investment attracted by the government always falls short of the amount H. It can be easily seen that otherwise $r \ge r_\mathrm{H}$ would hold, contrary to the previous finding. Hence, *the best choice of the government is to borrow less than amount H* (*which would minimize damage from a crisis*) *but at a lower than 'fair' interest rate*.

The consequence of using the 'hostage effect' is that only investors holding government debt will lend to the government. Indeed, it can be easily shown that if the i-th player holds no portfolio, his payoff under zero investment is zero, while under positive investment, it is always negative, *i.e.*, zero investment dominates any other decision in this case. The same holds if the portfolio is small enough.

Our next step is to demonstrate the importance of the 'hostage effect'. The most straightforward way to do this is to consider a simple version of the general game, having only two participants: the government and one investor. Suppose first that the investor holds a zero debt portfolio. One can easily see that in this situation only two solutions are possible: 1) a trivial solution (both suggested interest rate, and investment are zero), and 2) the government suggests rate r_H and investor lends amount H, under which the minimum level of losses is reached: $\omega_m = \omega(H)$.

The cooperative solution is realized if gains from minimizing crisis risk from $\omega(0)$ to $\omega(H)$ exceed costs of raising interest rates from

0 to r_H:

$$Z(\omega(0)) - Z(\omega_m) > \varphi(r_H) - \varphi(0).$$

The interest rate r_H can be characterized then as 'fair' as it fully covers lending risks. One can see that regardless of what type of solution is realized here, the payoff of the investor is zero, while the government may gain in the second case compared to the trivial solution. This means that the surplus from reducing crisis risk (if it exists) in the game with one investor bearing no portfolio is fully taken by the government.

Now we can discuss the more general feasibility of cooperation between the government and investor in crisis prevention. It makes sense to use here a 'zero option' as a benchmark when the government refrains from borrowing or investors decline lending. We try to find out under what conditions the government can suggest an interest rate which makes lending to the government attractive for an investor and the payoff of both of them becomes, as a result, higher than in the 'zero option.'

If the cooperative solution of the game (r^*, x^*) exists, then the following inequality holds:

$$r^* \geq \{r_0 + \omega(x^*) + d[\omega(x^*) - \omega(0)]/x^*\}/[1 - \omega(x^*)] \ . \qquad (3)$$

This ensures that lending is profitable for the investor. The first two terms on the right-hand side present just the standard requirement: the yield on investment should cover expected losses related to the pertinent risks. The inequality includes in addition a third term, which accounts for the specific incentives of the debt holders to evade a failure. Taking into account (as discussed above) that $x^* \leq H$, and keeping in mind that $\omega(x^*) > \omega_m$, we arrive at the following limitation for r^* from below:

$$r^* \geq [r_0 + \omega_m + d \min_{x \leq H} \omega'(X)]/(1 - \omega_m) =$$

$$= r_H + d \min_{x \leq H} \omega'(X)/(1 - \omega_m). \qquad (4)$$

Once $\omega(X)$ is a decreasing function in the range under consideration, debt held by investors relaxes the requirements for the yield suggested by the government: the interest rate may lie below the 'fair' level r_H (covering lending risks), if investors are already holding public debt.

On the other hand, putting together the preferences of the government and the investor, we can see that attracting investments makes sense for the government only if its gain from mitigating the crisis exceeds losses from raising the interest rate:

$$\varphi(r^*) + Z(\omega(x^*)) < \varphi(0) + Z(\omega(0)).$$

This inequality sets an upper limit on the possible range of yields suggested by the government: interest rate r^* should meet condition

$$\varphi(r^*) < \varphi(0) + Z(\omega(0)) - Z(\omega(H)) \quad (5)$$

(the right-hand side exceeds $\varphi(0)$, as losses are falling with investment growth from 0 to H).

Combining relationships (4) and (5), we obtain conditions when cooperation between the government and investors is impossible.

Proposition 2. If the adverse effect of raising interest rates is strong as compared to the effect of default,

$$\varphi(r_{\min}) - \varphi(0) > Z(\omega(0)) - Z(\omega(H)), \quad (6)$$

(where $r_{\min}$ denotes the right-hand side of (4)), then the game has only a trivial solution, *i.e.*, the government cannot reduce crisis damage, suggesting the investor's acceptable price for additional lending.

Indeed, it follows from (5) and (6) that

$$\varphi(r^*) \geq \varphi(r_{\min}) > \varphi(0) + Z(\omega(0)) - Z(\omega(H)),$$

which according to (4) means that lending to the government is not attractive for the investor. In other words, the ranges of interest rates acceptable for the government and for the investor have no intersection.

On the other hand, according to Proposition 1, the game has a positive solution if the debt portfolio is large enough. Inequality (4) elucidates the impact of the investor's debt holdings on the crisis development. According to (3), a higher debt portfolio diminishes the level of yield acceptable for the investor, and thus reduces the costs of mitigating the crisis for the government. This becomes possible because additional incentives of the 'old' investors to prevent debt failure allow the government to shift part of the costs on to these investors. We call this the 'hostage effect', as the government can use debt holdings to make investors bail out the government.

More than that, in some circumstances the 'hostage effect' can be the only chance for the government and investors to cooperate. Indeed, if the government does not make use of this effect in a game with one investor, it has to suggest at least a 'fair' yield r_H, acceptable to the investor regardless of his debt portfolio. It may well happen though that

$$\varphi(r_H) + Z(\omega(H)) > \varphi(0) + Z(\omega(0)),$$

and then cooperation is impossible as (5) cannot be met. At the same time we know that if the debt portfolio is large enough, the government can borrow, even suggesting a yield below the 'fair' level.

The 'hostage effect' reduces investors' payoff as compared to lending at the 'fair' yield. But, on the other hand, not only costs but also gains from mitigating a crisis are distributed between the government and investors. This follows from the fact (demonstrated in the proof of Proposition 1) that investors' payoff is growing with the amount of his lending in the range from zero to the optimum amount. This means, in particular, that using the 'hostage effect' in a situation where this is the only chance to cooperate, the government improves both its own payoff and that of investors.

6. GAME OF INVESTORS WITH FIXED YIELD

Further discussion requires, as an interim step, an in-depth analysis of the sub-game of investors with a given interest rate r. At the same time this analysis contributes to understanding the properties of the model as a whole.

We are considering below the sub-game of N investors with portfolios $(d_1, ..., d_N)$, assuming that the government has suggested some interest rate $r < r_H$. As a result, the focus of the analysis shifts from cooperation between the government and investor to coordination among investors.

Still, as in the previous section, the key point of the analysis is the role of the 'hostage effect'. Our assumption $r < r_H$ implies that the interest rate is set taking into account this effect.

Consequently, in line with the results of the previous section, the equilibrium lies in the range where $X < H$, *i.e.*, in the decreasing slope of function $\omega(X, R)$.

Below we investigate the 'comparative statics' of the sub-game solutions.

6.1. The effect of portfolio

It was proved above that if $(x_1, x_2, ..., x_N)$ is a positive Nash equilibrium in a sub-game A with interest rate r and portfolios d_i, investment made by different participants are linked by the following relations:

$$x_i + d_i/(1 + r) = x_j + d_j/(1 + r).$$

In other words, we obtain here rather unexpected result: though the only incentive to invest is to save a portfolio from depreciation, within the same positive equilibrium, the larger the portfolio, the less the investment. The reason is that the amount of investment is defined by a bal-

ance of the direct effect (interest less possible losses from devaluation) and the indirect effect (gain from a lower failure probability) from the unit of additional investment. The former effect is identical for all investors, hence the latter effect, proportional to total ('old' and 'new') assets, should be also identical in the Nash equilibrium.

On the other hand, participants with small portfolios do not come into the game at all, having zero new investment. This is evident from comparing the payoff function of a participant i in the arbitrary point $\{x^*\}=(x_1, x_2, ..., x_N)$, $x_i > 0$ and in the point x^0, which differs from x^* only in the i-th component, which equals zero here. Then

$$w_i^* - w_i^0 = [\omega(X-x_i) - \omega(X)]\, d_i + x_i \{r - r_0 - (1+r)\omega(X)\} <$$
$$< d_i[\omega(X-x_i) - \omega(X)] + x_i\rho(X).$$

The second term is negative (as $\rho(X) < r_H(1 - \omega_m) - r_0 - \omega_m = 0$, taking into account that we can restrict consideration with $r < r_H$); hence, if D is small, $w_i^* - w_i^0 < 0$, *i.e.*, $\{x^*\}$ is not a Nash equilibrium. On the other hand, if investment $\{x_k\}$, $k \neq i$ is an equilibrium in the game with $N - 1$ participants, then x^0 is also a Nash equilibrium in the initial game of N participants.

Thus we obtain that *the participant with the smallest portfolio either does not invest at all or makes the largest investment*. This is important. Next we will estimate the impact of the portfolio on investment.

Proposition 3. The total amount of investment in the game positively depends on the size of the portfolio held by each particular participant. On the other hand, the amount of investment made by the i-th participant negatively depends on his own portfolio (correspondingly, investment made by other participants positively depend on the i-th portfolio).

This is proved by estimating the derivative of equilibrium investment x_i and X by d_i, which can be obtained by differentiating equations (1) for all j by d_i. We get then

$$dX / d(d_i) = \frac{-\omega'}{-\sum w_{xx} - (N-1)(1+r)\omega'} > 0,$$
$$dx_i / d(d_i) = \frac{\sum w''_{xx} + (N-2)\omega'}{-\sum w''_{j\,xx} - (N-1)(1+r)\omega'} < 0. \qquad (7)$$

Applying second-order conditions, $w''_{xx} < 0$, and keeping in mind that in the case of negative real yield ω' is less than 0 at the equilibrium, we conclude that $dX/d(d_i) > 0$, $dx_i/d(d_i) < 0$.

If we restrain our consideration to symmetric games, the conclusion is that if all portfolios grow, the investment made by all participants increases.

6.2. Effect of reserves

We mentioned above that the government made efforts to increase liquidity by getting IMF loans. Now we can analyze the effect of these efforts in terms of our model.

Speaking informally, we assumed above that the declining slope of the 'loss function' ω reflects the short-term positive effect of additional financial resources, and increasing the slope of ω reflects the long-term negative impact of domestic borrowing. In the short run obtaining liquidity by borrowing abroad produces the same effect as borrowing domestically. In the long run there may be a difference related to the terms of these types of investment. But since we have demonstrated that the solutions are restricted within range $[0, r_H]$, we can disregard this difference. We introduce thus additional game parameter R — the amount of liquid reserves initially held by the government and assume that the latter are full substitutes for investment. In other words, payoffs of all participants depend on $\omega(X + R)$ instead of $\omega(X)$, as in the initial definition of the game.

The following result is surprising.

Proposition 4. The equilibrium amount of investment **negatively** depends on the size of the government's reserves.

Indeed, by differentiating the first order conditions by R, one obtains

$$X'_R = \frac{(1+r)\omega' + (D+X+Xr)\omega''}{(N+1)(1+r)\omega' + (D+X+Xr)\omega''} = -1 - \frac{N(1+r)\omega'}{\sum w''_j - (N-1)(1+r)\omega'} < 0, \qquad (8)$$

(where, as before, D is the total size of debt held by investors: $D = \sum d_i$).

An even stronger and more striking result holds: *an increase in reserves not only leads to a fall in total investment*, *but aggravates the crisis*. This is clear from analyzing the derivative of $(X + R)$ by R, which can be obtained from (8).

This means that not only the amount of investment, but also the total resources held by the government are declining as reserves are growing, *i.e.*, the negative reaction of investors on increase of government's reserves exceeds the size of this increase.

This unexpected result, which contradicts normal logic, is a consequence of the reverse nature of coordination in our game. One can easily see (by taking derivatives of the first-order conditions) that if the interest rate is fixed at some level below r_H and investment $\{x_2, ..., x_N\}$ by participants 2, ..., N are fixed, then the optimal amount of lending by the first participant x_1^* negatively depends on any of the variables x_2, ..., x_N. Contrary to the standard crises models, *larger support of the status quo by one investor creates disincentives to invest for others* instead of increasing incentives.

6.3. Number of participants

Let us analyze now the effect of changing the number of investors. We consider two games, one (A_1) with N, and another (A_2) with $N + 1$ identical investors (holding equal portfolios), both having positive solutions. It can be shown that if $w'(x)$ is decreasing in the range of x where $\omega(x)$ is decreasing, equilibrium investment x^* negatively depends on the number of participants. Indeed, let $(x_1, ..., x_1)$, and $(x_2, ..., x_2)$ be solutions in the games A_1 and A_2. Then $w'_N(x_1) = 0$, and $w'_{N+1}(x_2)=0$. Subtracting these equations we obtain

$$[w'_N(x_1) - w'_N(x_2)] + [w'_N(x_2) - w'_{N+1}(x_2)] =$$

$$= [w'_N(x_1) - w'_N(x_2)] + [w'_N(x_2) - w'_N(z)] -$$

$$- (1 + r)\,\omega'[(N + 1)x_2]\,(z - x_2) = 0,$$

where $z = (N + 1)/N\,x_2$.

Once $z > x_2$, both the second and the third items in the right part are positive. Hence the first item is negative, which is to say that $x_1 > x_2$.

This leads to a conclusion presented in the following proposition.

Proposition 5. Adding to the game A, an investor with the same portfolio d leads to a *decrease* in investment made by each investor.

In other words, coordination is deteriorating as the number of participants is growing. Combining this conclusion with the impact of a debt portfolio, we see that the '*hostage effect*' *shows itself best when the market is dominated by a limited number of large players*.

6.4. Effect of interest rate

Proposition 6. The total amount of investment in the game positively depends on the interest rate offered by the government.

Indeed, by differentiating the first order conditions by interest rate r, one obtains

$$\frac{dx}{dr} = \frac{1-(\omega(x)+x\omega'(x)]}{-w''_{xx}}.$$

As the denominator is positive (just as it was in (7)), and the numerator is evidently positive, we come to the conclusion that $dx/dr > 0$. Since the investment of each player positively depends on interest rate r, the total investment of all players will also positively depends on r.

7. ANALYSIS OF THE GENERAL GAME

Suppose now that the game solution implies positive investment. We consider below factors affecting the game outcome.

If game solution is positive, the maximum conditions of the first- and second order are met at the point (r^*, x^*):

$$(w_i)'_x(X, r) = (r - r_0) - (1+r)\omega(X) - (D + x_i + x_i r)\omega'(X) = 0, \quad (9)$$

$$(w_i)''_{xx}(X, r) = -2(1+r)\omega'(X) - (D + x_i + x_i r)\omega''(X) < 0.$$

The left side of (9), presenting the marginal payoff of the i-th investor, gives further insight into the nature of the 'hostage effect.' It consists of three parts: the first term reflects notional interest revenue from lending an additional unit (direct positive effect); the second term — losses due to pertinent risks of this lending (direct negative effect); and the third term — reduction of the investor's expected losses from the debt crisis due to additional lending to the government (indirect positive effect).

Proposition 7. If the game has a positive solution, the government's payoff is positively affected by an increase in the total portfolio size D.

Proof. Let (X^*, r^*) be a solution of the game.

The government's payoff is growing with X at the point $(X^*(r), r)$ under fixed r, if $X > X^*(r)$:

$$V(X) > V(X^*(r),r). \quad (10)$$

Indeed,

$$V(X) - V(X^*(r), r) = W(X^*(r), r) - W(X, r) + Z(\omega(X^*(r)) - Z(\omega(X)) > 0$$

as $W(X, r)$ reaches it maximum by X in the point $X^*(r)$, Z is growing with ω, and ω is decreasing by X at the solution point X^*.

Now, let $D_2 > D_1$. Taking into account Proposition 3 and relation (10), and denoting $r_1 = r^*(D_1)$, $r_2 = r^*(D_2)$ as the optimum rates chosen by the government given portfolio amounts D_1 and D_2, we obtain

$$V[X^*(r_2|D_2), r_2] > V[X^*(r_1|D_2), r_1],$$

as r_2 is the optimal yield when investor has portfolio D_2,

$$V[X^*(r_1|D_2), r_1] > V[X^*(r_1|D_1), r_1],$$

as, from the Proposition 3 we have $X^*(r_1|D_2) > X^*(r_1|D_1)$, and then (10) can be applied.

The impact of the portfolio on the interest rate and investment is still unclear: the government's payoff may increase either due to lower yield paid to the investor or due to larger investment and hence lower the risk of default.

The interpretation of Proposition 6 is apparent: the larger the portfolio held by the investor, the larger will be the contribution by investors to prevent a crisis.

The findings concerning the effect of higher reserves are more unexpected.

Proposition 8. If the government can gain by attracting investment, its payoff in the game is *negatively affected by an increase in the amount of reserves R*.

Proof. The proof of the Proposition 8 is similar to the proof of Proposition 7.

Let $R_2 > R_1$. Denoting $r_1 = r^*(R_1)$, $r_2 = r^*(R_2)$ as the optimum interest rates chosen by the investor given reserves R_1 and R_2, and taking into account Proposition 4, we obtain:

$$V[X^*(r_1|R_1), r_1] > V[X^*(r_2|R_1), r_2],$$

as r_1 is the optimal yield when investor has portfolio R_1,

$$V[X^*(r_2|R_1), r_2] > V[X^*(r_2|R_2), r_2] ,$$

as from the Proposition 4 we have $X^*(r_2|R_1) > X^*(r_2|R_2)$, and then (10) can be applied.

Again, it is unclear whether the government has to suggest a higher yield, gets less investment, or both as a result of higher reserves.

The situation is different if initially the game solution is 'trivial'. In this case an increase in reserves may raise incentives for the investor to participate and a positive solution may appear (the payoff of the government then increases). In other words, the effect of

raising government reserves may differ depending on the type of equilibrium. If initially cooperation was missing, higher reserves may make it possible, and the government gains from it. But if cooperation was already present, an increase in reserves makes the government worse off.

The conclusions of our analysis are robust, as the most important of them are based only on the most general assumptions of the model parameters (like the assumption that up to some level, the loss function negatively depends on the amount of investment).

8. CONCLUSION

We suppose that models presented in the previous sections can contribute to understanding the 'government-investors' relationship in a course of an acute crisis.

First, we have shown that the government can use debt held by investors as a 'hostage' and make them incur part of the crisis costs. Second, in the most severe situations this turns out to be the only way for the government (or project manager) to attract additional investment and thus to mitigate the crisis. Third, we found that in this model, the more reserves the government (or manager) has, the stronger is the adverse effect of the crisis. If, according to normal logic, additional government reserves should raise investor's confidence to the government and thus result in cheaper borrowing, in our model quite the opposite is true: additional reserves lead to less bail-out efforts by investors and hence higher yields and/or less investment.

Our game can be regarded as a model of 'mandatory cooperation' between the government and investors in crisis prevention. On the other hand, the coordination among investors realized in the game has a reverse nature: the larger the lending by one investor, the less others lend. The same effect provides additional liquidity by international financial institutions, hence a category of situations where IMF loans adversely affect a crisis is identified.

We assume also that these models contribute to understanding the mechanics of the final stage of the Russian financial crisis (its 'end-game'), as the facts on the development of the financial crisis in Russia show that at some stage, the situation could correspond to the one introduced in the game considered above. The common features of the model and the actual situation include also:

- addressing the crisis as 'transient' by the monetary authorities;

- investors' expectations of substantial loss in the real value of government bonds (yields at the secondary market exceeded 100% in some periods);
- high concentration of the GKO/OFZ market, still aggravated in the course of the crisis;
- the outburst of the crisis in August 1998, immediately after the disbursement of a large loan by international financial institutions.

Our model may shed light on the causes of the August 1998 crisis and on receiving additional reserves from the IMF and the World Bank. Our hypothesis is that an increase in reserves resulted in lower investment in the GKO market; this is in line with the findings of the model.

The following policy recommendations can be derived from our analysis.

- In fighting a crisis, the government has to realize clearly in what area the situation is. It may happen that evaluation of the risk by the government differs from that by the investors. The former may suggest, from its viewpoint, a fair interest rate, which the latter will accept due to the 'hostage effect' (though the latter will regard the rate as lower than 'fair'). In this case the standard measures taken by the government, like getting an IMF loan, may have adverse effect, opposite than expected.
- If the government deliberately resorts to the 'hostage effect', it should be fully aware of non-standard reactions to some standard measures in this case, that is, those concerned with reverse coordination.
- It is crucial for the 'creditor of last resort' to distinguish the situation of 'reverse coordination' from that of standard coordination, as in the former case its loans may turn out to be harmful, thus aggravating the crisis.
- The presence of large players may stabilize the market by making it possible for the government to use the 'hostage effect' and reinforcing the hostage effect if it is used.

While some models demonstrate how rational behavior may result in a crisis, we give an example of a crisis fueled by actions that look rational from the common sense viewpoint, but have the opposite effect than what is expected.

Using the 'hostage effect' may have adverse long-term implications, which are not reflected in our model — say, in terms of reputation. But we may assume that the damage from default reflects the difference between reputation damages contributed to defaulting and to using the 'hostage effect'.

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